Scaling properties of proton-nucleus total reaction cross sections

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We study the scaling properties of proton-nucleus total reaction cross sections for *stable* nuclei and propose an approximate expression in proportion to $Z^{2/3}\sigma_{pp}^{\rm total} + N^{2/3}\sigma_{pn}^{\rm total}$. Based on this expression, we can derive a relation that enables us to predict a total reaction cross section for any stable nucleus within 10% uncertainty at most, using the empirical value of the total reaction cross section of a given nucleus.

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Recently, we studied total reaction cross sections of carbon isotopes ($N = 6-16$) incident on a proton for a wide energy region [1]. In that work, using our numerical results, we found an empirical formula, Eq. (13) of Ref. [1] (see the Appendix). In this Brief Report, as an extension of the empirical formulae, we study the scaling properties of proton-nucleus total reaction cross sections for *stable* nuclei of the whole mass-number region. Our new expression is found to be in proportion to $Z^{2/3}\sigma_{pp}^{\rm total} + N^{2/3}\sigma_{pn}^{\rm total}$. Although this is merely a parameterization without any microscopic foundation, once one is able to accept this expression, we can derive a new relation (4) that enables us to predict a total reaction cross section for any stable nucleus using only the empirical values of the total reaction cross section of a given nucleus at a given energy within 10% uncertainty.

We assume here the following expression for the total reaction cross sections of a proton incident on a nucleus with the mass number, $A(= N + Z)$:

$$\begin{aligned} & \sigma_{\rm R}(Z,N,E) \\ = \; & \pi C(E,A)\{Z^{2/3}\sigma_{pp}^{\rm total}(E) + N^{2/3}\sigma_{pn}^{\rm total}(E)\} \\ \simeq \; & \pi C(E) A^{2/3} \times \\ & \Big[(Z/A)^{2/3}\sigma_{pp}^{\rm total}(E) + (N/A)^{2/3}\sigma_{pn}^{\rm total}(E)\Big], \quad (1) \end{aligned}$$

where $Z(N)$ is the number of protons (neutrons) in the target nucleus, $\sigma_{pp}^{\rm total}$ ($\sigma_{pn}^{\rm total}$) is the proton-proton (proton-neutron) total cross section, and $C(E)(\equiv \langle C(E,A)\rangle_A)$ is an energy-dependent coefficient to be deduced from data. $\langle\cdots\rangle_A$ implies the average value of the values for different mass numbers at each energy. Note that Eq. (1) is merely a parameterization without any microscopic foundation and that the factorization of $C(E)$ is crucial for later discussion. This expression can predict the energy dependence of total cross sections of lead ($Z = 82$, $N = 126$) with acceptable uncertainties, but, unfortunately, the isospin dependence is not necessarily adequate for describing neutron-rich unstable nuclei.

In Eq. (1), $C(E,A)$ in the second line is replaced by $C(E)$, because $C(E,A)$ depends weakly on A. Figure 1 shows the values of $\pi C(E,A)$ as functions of the mass number for all available nuclei at six different energies [2]. The solid lines in each panel represent the values of $\pi C(E)$ obtained by averaging $\pi C(E,A)$ over the whole mass numbers for each energy. The dashed lines denote the standard deviations of $\pi C(E)$. These results validate the replacement of $C(E,A)$ with $C(E)$ in Eq. (1).

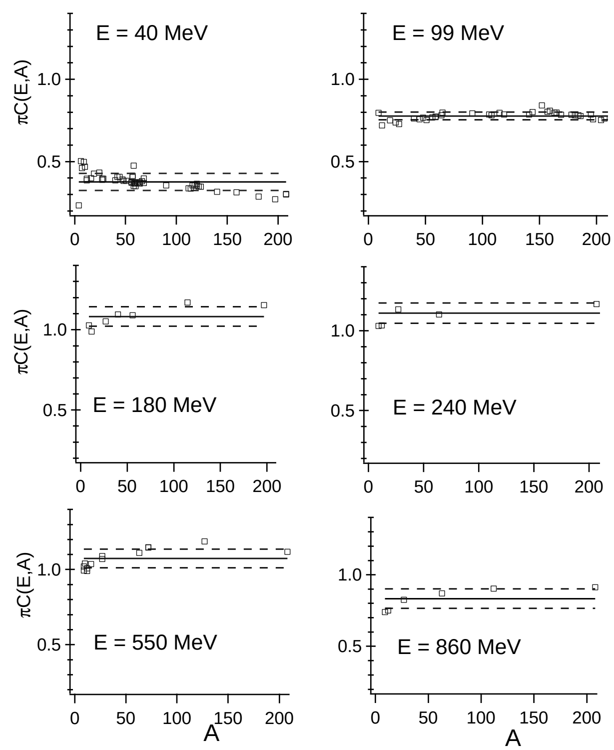

FIG. 1: Values of $\pi C(E,A)$ as functions of target mass number, A, for each energy, E, indicated by open squares. For convenience, we plot $\pi C(E,A)$ instead of $C(E,A)$. The solid lines indicate the average values. The dashed lines indicate standard deviations around each average value. The experimental data are taken from Ref. [2].

In Fig. 2, we summarize the values of $\pi C(E)$ as a function of energy. Those values are also listed in Table I with

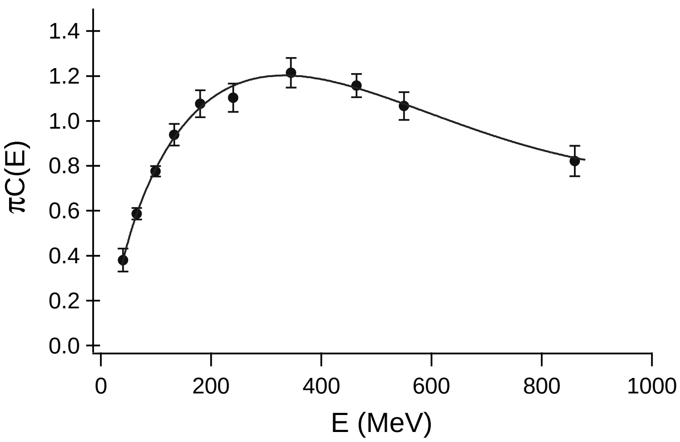

FIG. 2: Values of coefficient, $\pi C(E)$, as functions of proton incident energy, E. The solid curve shows the behavior of Eq. (3).

their standard deviations defined by

$$\Delta C(E) = \sqrt{\frac{1}{N_{\rm data}} \sum_{i=1}^{N_{\rm data}} (C_i - \bar{C})^2}. \tag{2}$$

To facilitate the numerical calculations of $\sigma_{\rm R}$ using Eq. (1), it would be useful to prepare the analytical form for $C(E)$ as a function of proton incident energy, E. We adopt, for example,

$$\begin{aligned} \pi C(E) &= a_1 - a_2 \exp(-a_3 E^{a_4}) \\ &\times \cos[a_5 E^{a_6}], \end{aligned} \tag{3}$$

where E is the projectile kinetic energy in units of MeV. The constants are given by a_1=0.909824, a_2=2.329619, a_3=0.4345765, a_4=0.2655287, a_5=0.075, and a_6=0.6262386. The behavior of Eq. (3) is represented by the solid curve in Fig. 2.

Using the preceding setups, in Fig. 3 we show the results of the formula with the data of $\sigma_{\rm R}$ as a function of mass number, A, at two different energies. The solid curves show the results of Eq. (1). The values of the coefficient, $\pi C(E)$, that provide the solid curves are listed in Table I. The agreement of the numerical results with the empirical data is fairly good. Although we do not show the results for $E = 40$ MeV, one should be careful in the application of this formula at $E = 40$ MeV because of the large deviation from the data for $A \geq 100$.

TABLE I: Values of coefficient, $\pi C(E)$, defined in Eq. (1), and their standard deviations. The curves in Fig. 3 are drawn using these values.

E (MeV)	$\pi C(E)$	E (MeV)	$\pi C(E)$
40	0.3764±0.051	240	1.1104±0.063
65	0.5868±0.025	345	1.2144±0.066
99	0.7776±0.023	464	1.1570±0.052
133	0.9384±0.048	550	1.0741±0.062
180	1.0822±0.060	860	0.8334±0.068

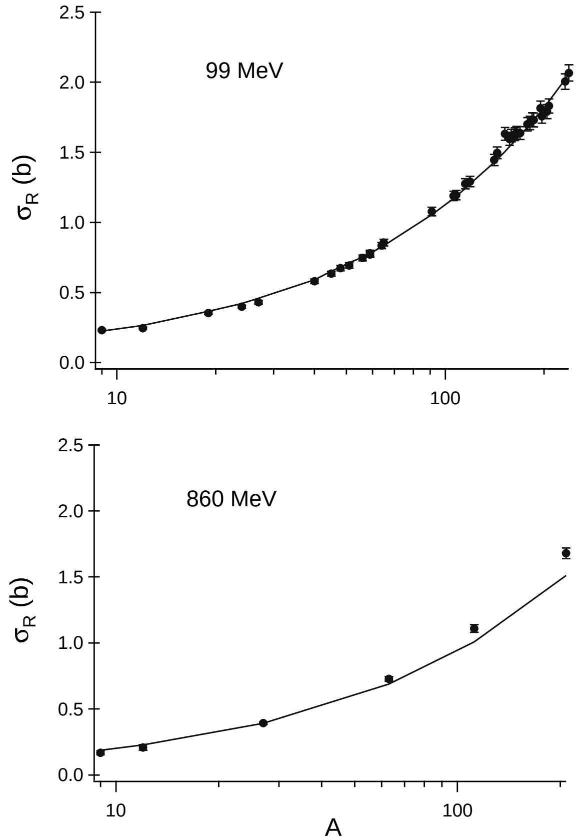

FIG. 3: Comparison of the numerical results obtained using Eq.(1) with the empirical data of total reaction cross sections for proton-nucleus reactions as a function of mass number, A, for E = 99 and 860 MeV. The solid curves are the numerical results. The values of $\pi C(E)$ are listed in Table I. The experimental data are taken from Ref. [2].

Also, we show the results of the formula with the empirical data of $\sigma_{\rm R}$ as a function of proton incident energies in Fig. 4. The solid curves represent the numerical results of Eqs. (1) and (3) for the total reaction cross section of protons incident on two target nuclei, such as C and Pb.

To show the precision of our formula, Eq. (1), we estimate the fluctuations of the data around the numerical results at each energy. We introduce the fluctuation defined by the standard deviations of $C(E)$ divided by its mean values, that is, $\Delta C(E)/C(E)$, and plot them as a function of energy, E, in Fig. 5. One can see that, except for the case of 40 MeV, the uncertainties in our numerical results are less than 10%. That is, the feature of each nucleus affects the magnitude of the total reaction cross section by 10% at most.

Let us derive the "parameter-free" expression of $\sigma_{\rm R}$ us-

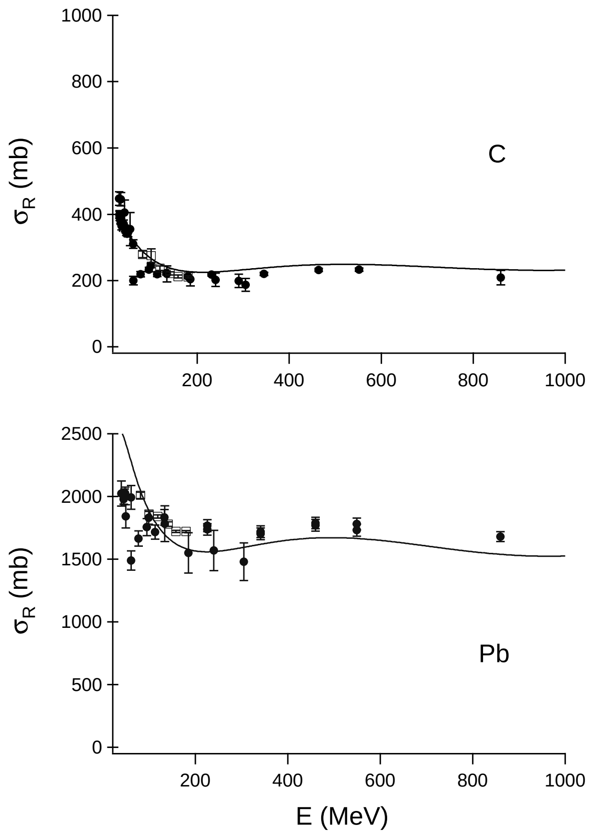

FIG. 4: Comparison of the numerical results obtained using Eqs.(1) and (3) with the empirical data of the total reaction cross sections for proton-nucleus reactions as a function of energy, E. The solid curves are the numerical results of Eqs. (1) and (3). The experimental data are taken from Ref. [2] (solid circles) and Ref. [3] (open squares).

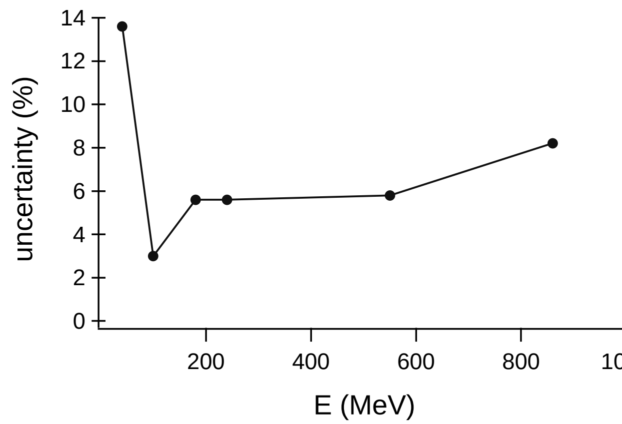

FIG. 5: Uncertainties in numerical results of $\sigma_{\rm R}$ of Eq. (1) as functions of proton incident energy, E. The solid line is a guide for the eye.

ing Eq. (1), which is the major topic of this article. In Eq. (1), we introduced the coefficient, $C(E)$ as being independent of A within 10% uncertainty. If we accept this fact, we can estimate a total reaction cross section of a nucleus $A_1(=Z_1+N_1)$ at a given energy using a known value of the total reaction cross section of another nucleus $A_2(=Z_2+N_2)$ at the same energy. The expression becomes

$$\sigma_{\rm R}(Z_1,N_1) = (1\pm\Delta)\left(\frac{\sigma_{pp}Z_1^{2/3}+\sigma_{pn}N_1^{2/3}}{\sigma_{pp}Z_2^{2/3}+\sigma_{pn}N_2^{2/3}}\right)\sigma_{\rm R}(Z_2,N_2), \tag{4}$$

where Δ implies the uncertainty coming from that of $C(E)$. Here, Δ is defined by $\Delta \equiv \langle\delta(E)\rangle_E$, and $\delta(E)\equiv 2\Delta C(E)/C(E)$. $\langle\delta(E)\rangle_E$ implies the average of $\delta(E)$ values at various energies. We obtain $\Delta \simeq 0.1$. No fitting parameter appears here, but, at most, 10% uncertainty should be taken into account.

In Fig. 6, we show the numerical results of Eq. (4) with the empirical data as a function of mass number, A, at 99 and 860 MeV. For the reference nucleus, which corresponds to A_2 in Eq. (4), we here choose a nucleus of a medium mass number, such as ^{56}Fe, ^{64}Zn, and ^{64}Cu. For numerical calculations of Eq. (4) for 40, 99, 180, 550 and 860 MeV, we choose a nucleus and the reaction cross section at each energy as (^{64}Zn, 1092±23 mb), (^{64}Cu, 835±23 mb), (^{56}Fe, 662±19 mb), (^{64}Cu, 667±67 mb), (^{64}Cu, 777±17 mb) and (^{64}Cu, 728±17 mb), respectively [2]. Here we neglect Δ.

The expression (4) indicates that, if we have the empirical value of the reaction cross section of a given nucleus, we can predict the reaction cross section of other nuclei. The numerical results are consistent with the empirical data of stable nuclei. A possible way of estimating an unknown reaction cross section of a given nucleus at a given energy is to use Eqs. (1) and (3) or to use Eq. (4). We do not need any parameter for the latter. The fluctuations in $\sigma_{\rm R}$ coming from the feature of each nucleus affects the magnitude by 10% at most.

In summary, we have introduced an approximate but new expression for proton-nucleus total reaction cross sections in terms of the number of protons and neutrons, and the total cross sections of proton-proton and proton-neutron reactions. This expression is applicable in the energy range from 40 MeV to 1000 MeV. Also, we have derived a simple relation for predicting the total reaction cross sections of a proton with any nucleus within 10% uncertainty, using the empirical values of the total reaction cross section of a given nucleus at a given energy. A better result is obtained if a neighboring nucleus is adopted.

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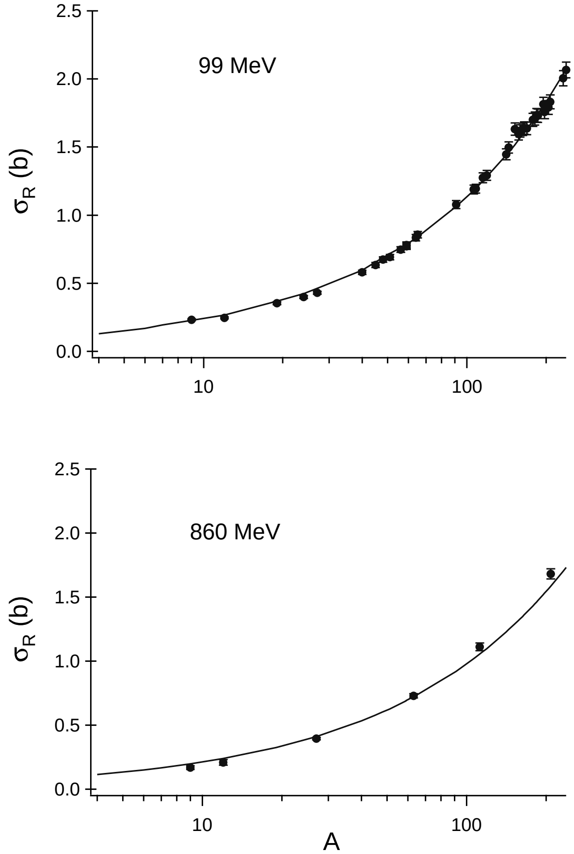

FIG. 6: Comparison of the numerical results of Eq. (4) with the empirical data of the total reaction cross sections for proton-nucleus reactions as functions of mass number, A. The solid curves represent the numerical results obtained using Eq. (4). The experimental data are taken from Ref. [2].

Appendix A: Volume-type form

We found that, in Eq. (13) of Ref. [1], for all the carbon isotopes, the following relation is satisfied over the entire energy range:

$$\frac{\sigma_{\mathrm{R}}(p+{}^{6+N}_{\ \ 6}\mathrm{C})}{\sigma_{\mathrm{R}}(p+{}^{12}_{\ 6}\mathrm{C})} = R(\mathrm{C})\frac{6\sigma_{pp}^{\mathrm{total}}+N\sigma_{pn}^{\mathrm{total}}}{6\sigma_{pp}^{\mathrm{total}}+6\sigma_{pn}^{\mathrm{total}}}, \tag{A1}$$

where $R(\mathrm{C}) = 0.96\pm0.05$ and $N \geq 7$. The above relation suggests the following volume-type formula:

$$\sigma_{\mathrm{R}}(Z,N,E) = C(E)\left[Z\sigma_{pp}^{\mathrm{total}}(E)+N\sigma_{pn}^{\mathrm{total}}(E)\right]. \tag{A2}$$

We find that this formula is able to estimate the proton-nucleus cross section in the mass number range

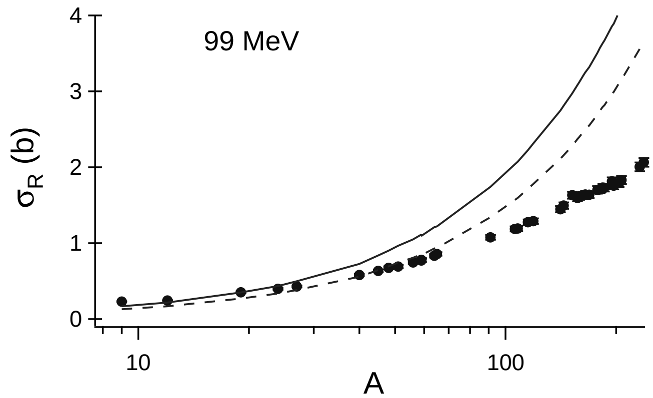

FIG. 7: Prediction using Eq. (A2) for the proton-nucleus total reaction cross sections at 99 MeV. The nuclei with $4 < A < 30$ fit the results with $C(E) = 0.3523198$ (solid curve) and those with $30 < A < 60$ fit the results with $C(E) = 0.27075$ (dashed curve).

around $b < A < b+30$, where b is any number. The applicability is limited to a certain restricted region in A. Figure 7 shows the numerical results of Eq. (A2). The nuclei with $4 < A < 30$ fit the results with $C(E) = 0.3523198$ (solid curve) and those with $30 < A < 60$ fit the results with $C(E) = 0.27075$ (dashed curve).

[1] B. Abu-Ibrahim, W. Horiuchi, A. Kohama and Y. Suzuki, Phys. Rev. C **77**, 034607 (2008); Erratum: *ibid.* C **80**, 029903 (2009).

[2] R. F. Carlson, At. Data and Nucl. Data Tables **63**, 93 (1996).

[3] A. Auce, A. Ingemarsson, R. Johansson, M. Lantz *et al.*, Phys. Rev. C **71**, 064606 (2005).